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Differential Calculus (Curvature)

Q.1 Show that the chord of curvature through the pole of the curve $r = a e^{m\theta}$ is $2r$.

Soln. We have $r_1 = am e^{m\theta} = mr$ and $r_2 = am^2 e^{m\theta} = m^2 r$

The chord of curvature through the pole

$$= \frac{2r(r^2+r_1^2)}{r^2+2r_1^2 - rr_2} = \frac{2r\cancel{r^2}\cancel{(1+m^2)}}{\cancel{r^2}\cancel{(1+m^2)}} = 2r$$

Q.2. In the lemniscate $r^2 = a^2\cos 2\theta$, show that the chord of curvature through the origin is $\frac{2}{3}r$.

Soln. We have, $r^2 = a^2\cos 2\theta$ —— (I)

Let us first of all we find its pedal equation,

Differentiating (I) with respect to θ, we get

$$2r\frac{dr}{d\theta} = -2a^2\sin 2\theta \Rightarrow \frac{d\theta}{dr} = -\frac{r}{a^2\sin 2\theta}$$

$$\text{But } \tan\phi = r\frac{d\theta}{dr} = -\frac{r^2}{a^2\sin 2\theta} = -\frac{a^2\cos 2\theta}{a^2\sin 2\theta} = -\cot 2\theta$$

$$\Rightarrow \tan\phi = \tan\left(\frac{\pi}{2}+2\theta\right) \Rightarrow \phi = \frac{\pi}{2}+2\theta \text{ —— (II)}$$

We have, $p = r\sin\phi = r\sin\left(\frac{\pi}{2}+2\theta\right) = r\cos 2\theta = r\times\frac{r^2}{a^2}$ [by I]

$$\therefore \Rightarrow p = \frac{r^3}{a^2}$$

$$\therefore \frac{dp}{dr} = \frac{3r^2}{a^2}$$

Hence, the chord of curvature through the origin

$$= 2p\frac{dr}{dp} = 2\frac{\cancel{r^3}}{\cancel{a^2}}\times\frac{\cancel{a^2}}{3\cancel{r^2}} = \frac{2}{3}r.$$

Proved

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Q.3. Show that the chord of curvature through the pole of the curve

$$r^m = a^m \cos m\theta \text{ is } \frac{2r}{m+1}$$

Soln: It is given that $r^m = a^m \cos m\theta$

Taking logarithm on both sides, we get

$\log r^m = \log(a^m \cos m\theta)$

$\Rightarrow m \log r = \log a^m + \log \cos m\theta$

$\Rightarrow m \log r = m \log a + \log \cos m\theta$

Differentiating w.r.t. θ, we get

$\therefore \frac{m}{r}\frac{dr}{d\theta} = 0 - m\tan m\theta = -m\tan m\theta$

$\Rightarrow r\frac{d\theta}{dr} = -\cot m\theta = \tan\left(\frac{\pi}{2} + m\theta\right)$

$\because \tan\phi = r\frac{d\theta}{dr} = \tan\left(\frac{\pi}{2} + m\theta\right)$

$\therefore \phi = \frac{\pi}{2} + m\theta$

$\because p = r\sin\phi = r\sin\left(\frac{\pi}{2} + m\theta\right) = r\cos m\theta$

$\Rightarrow p = r \cdot \frac{r^m}{a^m} = \frac{r^{m+1}}{a^m}$

$\therefore \frac{dp}{dr} = \frac{(m+1)r^m}{a^m}$

The chord of curvature through the pole

$$= 2p\frac{dr}{dp} = 2 \cdot \frac{r^{m+1}}{a^m} \times \frac{a^m}{(m+1)r^m} = \frac{2r}{m+1}$$

Q.4. Prove that the chord of curvature parallel to the axis of y for the curve $y = a\log\sec\frac{x}{a}$ is of constant length.

Soln: We have, $y_1 = a \cdot \frac{1}{\sec\frac{x}{a}} \sec\frac{x}{a} \cdot \tan\frac{x}{a} \cdot \frac{1}{a} = \tan\frac{x}{a}$

$y_2 = \frac{1}{a}\sec^2\frac{x}{a} \quad \therefore \rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2} = \frac{a\sec^3\frac{x}{a}}{\sec^2\frac{x}{a}} = a\sec\frac{x}{a}$

Now, $\tan\psi = \frac{dy}{dx} = y_1 = \tan\frac{x}{a}$

$\therefore \cos\psi = \frac{1}{\sqrt{1+\tan^2\psi}} = \cos\frac{x}{a}$

The chord of curvature parallel to the axis of y

$$= 2\rho\cos\psi = 2a\sec\frac{x}{a} \cdot \cos\frac{x}{a} = 2a = \text{constant.}$$

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